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A NOTE ON BAILEY'S TRANSFORM AND HYPERGEOMETRIC FUNCTIONS

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ABSTRACT

In this article, we have made an attempt to establish certain summation formulae for Hypergeometric functions by making use of Bailey's transform and known summation formulae for truncated series.

Mathematics Subject Classification:
Keywords: Hypergeometric functions, Bailey's transform, Truncated series, Summation formulae.

1. INTRODUCTION

Bailey in 1947 established a very beautiful and useful transform in the following form:

if $\beta_n = \sum_{r=0}^{n} \alpha_r u_{n-r} v_{n+r}$ and

$$\gamma_n = \sum_{r=n}^{\infty} \delta_r u_{r-n} v_{n+r}, \qquad (1.1)$$

where α_r, δ_r, u_r and v_r are any functions of r alone such that γ_n exists, then

$$\sum_{n=0}^{\infty} \alpha_n \gamma_n = \sum_{n=0}^{\infty} \beta_n \delta_n, \qquad (1.2)$$

under suitable condition of convergence.

Bailey's contributions in the field of transformations theory provided a new platform for the mathematician working in the field of Ramanujan's mathematics. Bailey's [2] used these transformations to obtain a number of Rogers-Ramanujan type identities and subsequently using the same transformation Slater, L. J. [6] gave a long list of 130 identities. Recently Singh, U. B.[5], Srivastava Pankaj[7], srivastava Pankaj and Mohan R.[8] added some new flowers in the garden of Bailey's transform. In the present paper we establish certain summation formulae for Hypergeometric functions by making use of Bailey's transform and known summation formulae due to Verma, A. and Jain, V. K. [9] and Denis, R. Y. [3,4].

2. DEFINITIONS AND NOTATIONS

$${}_2F_1\left[\begin{matrix} a, b; z \\ c \end{matrix}\right] = \sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{(c)_n n!} z^n, \qquad (2.1)$$

where a, b and c are complex parameters.

The Factorial function is:

$$(a)_n = a(a+1)(a+2)\ldots(a+n-1). \tag{2.2}$$

We shall use the following known summation formulae:

$${}_3F_2\left[\begin{matrix} x,\ 3x+4+n,-n \\ \frac{3}{2}x+\frac{3}{2},\frac{3}{2}x+2 \end{matrix};3/4\right] = \frac{(1)_n(2x+4)_n(x+2)_m(x+3)_{3m}}{(1+x)_n(3x+4)_n(1)_m(2x+4)_{3m}}. \tag{2.3}$$

$${}_3F_2\left[\begin{matrix} -n,\ x,y \\ -n-x,-n-y \end{matrix}\right] = \frac{(1)_n(1+x+y)_n(1+x)_m(1+y)_m}{(1+x)_n(1+y)_n(1)_m(1+x+y)_m}. \tag{2.4}$$

$${}_3F_2\left[\begin{matrix} -n,-n-x,y \\ 1+x,-n-y \end{matrix}\right] = \frac{(1)_n(1+x-y)_n(1+y)_m}{(1+x)_n(1+y)_n(1)_m}. \tag{2.5}$$

$${}_3F_2\left[\begin{matrix} -n,\ 1+x,1+y \\ 1-n-x,1-n-y \end{matrix}\right] = \frac{(-1)^n(1)_n(1+x+y)_n(1+x)_m(1+y)_{3m}}{(x)_n(y)_n(1)_m(1+x+y)_m}. \tag{2.6}$$

$${}_3F_2\left[\begin{matrix} -n,\ -n-2x,\ y \\ -n-x,\ 2y+1 \end{matrix}\right] = \frac{(1)_n(1+x+y)_n(1+x)_m(1+y)_m}{(1+x)_n(1+2y)_n(1)_m(1+x+y)_m}. \tag{2.7}$$

$${}_3F_2\left[\begin{matrix} -n,\ -n-2x,1+y, \\ 1-n-x,2y+1 \end{matrix}\right] = \frac{(-1)^n(1)_n(1+x+y)_n(1+x)_m(1+y)_m}{(x)_n(1+2y)_n(1+x+y)_m(1)_m}. \tag{2.8}$$

$${}_3F_2\left[\begin{matrix} -n,\ 1+n+2x+2y,\ x \\ 1+x+y,1+2x \end{matrix}\right] = \frac{(1)_n(1+x)_m(1+y)_m}{(1+2x)_n(1+x+y)_m(1)_m}. \tag{2.9}$$

$${}_3F_2\left[\begin{matrix} -n,\ 1+n+2x+2y,1+x \\ 1+x+y,1+2x \end{matrix}\right] = \frac{(-)^n(1)_n(1+x)_m(1+y)_m}{(1+2x)_n(1+x+y)_m(1)_m}. \tag{2.10}$$

$${}_3F_2\left[\begin{matrix} -n,\ 2+b+n+2x,1+x \\ 1+\frac{1}{2}b+x,2+2x \end{matrix}\right] = \frac{(-)^n(1)_n\left(\frac{3}{2}+\frac{b}{2}+x\right)_m\left(1+\frac{b}{2}\right)_m}{(2+b+2x)_n(1)_m\left(\frac{3}{2}+x\right)_m}. \tag{2.11}$$

$${}_4F_3\left[\begin{matrix} x,\frac{1}{2}x-\frac{1}{2},1+a+n,-n \\ x-1,x+1,\frac{1}{2}+\frac{1}{2}a \end{matrix}\right] = \frac{(1)_n(1+a-x)_n\left(1+\frac{1}{2}a\right)_m\left(\frac{1}{2}+\frac{1}{2}a\right)_m}{(1+a)_n(1+x)_n(1)_m\left(\frac{1}{2}+\frac{1}{2}a-\frac{1}{2}x\right)_m}. \tag{2.12}$$

3. MAIN RESULTS:

The main results are follows:

$$\frac{1}{(1-z)^{3x+4}}\,{}_3F_2\left[\begin{matrix} x, 3x+4, 3x+4+n \\ \\ \frac{3}{2}x+\frac{3}{2}, \frac{3}{2}x+2 \end{matrix} ; \frac{\frac{-3z}{4}}{(1-z)^2}\right] = \sum_{n=0}^{\infty} \frac{(2x+4)_n (x+2)_n (x+3)_{3m}}{(1+x)_n (1)_m (2x+4)_{3m}} z^n. \tag{3.1}$$

$$\frac{\Gamma(\alpha)\Gamma(\alpha-x-y-2)}{\Gamma(\alpha-x-1)\,\Gamma(\alpha-y-1)}\,{}_3F_2\left[\begin{matrix} x, y, \alpha-x-y-2 \\ \alpha-x-1, \alpha-y-1 \end{matrix}\right] =$$

$$\sum_{n=0}^{\infty} \frac{(1+x+y)_n (1+x)_m (1+y)_m}{(1)_n (1+x+y)_m} \frac{1}{(\alpha)_n} \tag{3.2}$$

$$\frac{\Gamma(1+x)\Gamma(x-y-\alpha)}{\Gamma(x-y)\,\Gamma(1+x-\alpha)}\,{}_3F_2\left[\begin{matrix} \alpha, \alpha-x, y \\ 1+x, 1-x+y+\alpha \end{matrix}\right] =$$

$$\sum_{n=0}^{\infty} \frac{(\alpha)_n (1+x-y)_m (1+y)_m}{(1+x)_m (1)_m (1+x)_m}. \tag{3.3}$$

$$\frac{\Gamma(z)\Gamma(z-x-y)}{\Gamma(x-y)\,\Gamma(1+x-\alpha)}\,{}_3F_2\left[\begin{matrix} 1+x, 1+y, z-x-y \\ ;1 \\ z-x, z-y \end{matrix}\right] =$$

$$\sum_{n=0}^{\infty} \frac{(1+x+y)_n (1+x)_m (1+y)_m}{-(z)_n (1)_m (1+x+y)_m}. \tag{3.4}$$

$${}_3F_2\left[\begin{matrix} y, \alpha, x-\alpha \\ \\ 2y+1, 1+2x-\alpha \end{matrix} ;-1\right]$$

$$= \frac{\Gamma(x)\Gamma(1+2x-\alpha)}{\Gamma(1+2x)\,\Gamma(x-\alpha)} \sum_{n=0}^{\infty} \frac{(1+x+y)_n (\alpha)_n (1+x)_m (1+y)_m}{(1+2x)_n (1+2y)_n (1+x+y)_m}. \tag{3.5}$$

$$\frac{\Gamma(1+2x)\Gamma(1+x-\alpha)}{\Gamma(1+x)\,\Gamma(1+2x-\alpha)}\ {}_2F_2\begin{bmatrix}1+y,1+x-\alpha\\1+2y,1+2x-\alpha\end{bmatrix}$$

$$=\sum_{n=0}^{\infty}\frac{(1+x+y)_n(1+x)_m(1+y)_m}{(1+2x)_n(1+2y)_n(1+x+y)_m(1)_m}. \qquad (3.6)$$

$$\frac{1}{(1-z)^{1+2x+2y}}\ {}_3F_2\begin{bmatrix}x,1+2x+2y,1+2x+2y+n & \\ & ;z/(z-1)^2\\ 1+x+y,1+2x & \end{bmatrix}$$

$$=\sum_{n=0}^{\infty}\frac{(z)^n(1+2x+2y)_n(1+x)_m(1+y)_m}{(1+2x)_n(1+x+y)_m(1)_m}. \qquad (3.7)$$

$$\frac{1}{(1-z)^{1+2x+2y}}\ {}_3F_2\begin{bmatrix}1+x,1+2x+2y,1+2x+2y+n & \\ & ;-z/(z-1)^2\\ 1+x+y,1+2x & \end{bmatrix}$$

$$=\sum_{n=0}^{\infty}\frac{(1+2x+2y)_n(-z)^n(1+x)_m(1+y)_m}{(1+2x)_n(1+x+y)_m(1)_m}. \qquad (3.8)$$

$${}_3F_2\begin{bmatrix}1+x,\,2+b+2x,\,2+b+2x+n\,;\frac{-z}{(1-z)^2}\\ 1+\frac{1}{2}b+x,2+2x\end{bmatrix}$$

$$=(1-z)^{2+b+2x}\sum_{n=0}^{\infty}\frac{(\frac{3}{2}+\frac{1}{2}b+x)_m(1+\frac{1}{2}b)_m}{(1)_m(\frac{3}{2}+x)_m}(-z)^n. \qquad (3.9)$$

$${}_4F_3\begin{bmatrix}x,\frac{1}{2}x-\frac{1}{2},1+a,1+a+n & \\ & ;\frac{-z}{(1-z)^2}\\ x-1,\,x+1,\frac{1}{2}+\frac{a}{2} & \end{bmatrix}$$

(3.10)

$$=(1-z)^{1+a}\sum_{n=0}^{\infty}\frac{(1+a-x)_n z^n(1+\frac{1}{2}a)_m(\frac{1}{2}+\frac{1}{2}a)_m}{(1+x)_n(1)_m(\frac{1}{2}+\frac{1}{2}a-\frac{1}{2}x)_m}.$$

4. PROOF OF MAIN RESULTS

(i) As an illustration, we prove (3.1).

Taking $u_r = \frac{1}{(1)_r}$, $v_r = (3x+4)_r$, $\alpha_r = \frac{(x)_r(-1)_r\left(\frac{3}{4}\right)^r}{(\frac{3}{2}x+\frac{3}{2})_r(\frac{3}{2}x+2)_r(1)_r}$ and $\delta_r = z^r$ in (1.1) we get:

$$\beta_n = \sum_{r=0}^{n} \frac{(x)_r(-1)_r\left(\frac{3}{4}\right)^r}{(\frac{3}{2}x+\frac{3}{2})_r(\frac{3}{2}x+2)_r(1)_r} \frac{1}{(1)_{n-r}} (3x+4)_{n+r}$$

$$= \sum_{r=0}^{n} \frac{(x)_r(3x+4+n)_r(-n)_r\left(\frac{3}{4}\right)^r}{(\frac{3}{2}x+\frac{3}{2})_r(\frac{3}{2}x+2)_r(1)_r} \frac{(3x+4)_n}{(1)_n}$$

Now, making use of (2.3), we get:

$$= \frac{(2x+4)_n (x+2)_m (x+3)_{3m}}{(1+x)_n (1)_m (2x+4)_{3m}}$$

$$\gamma_n = \sum_{r=0}^{\infty} z^{r+n} + n\frac{1}{(1)_r}(3x+4)_{r+2n}$$

$$= (3x+4)_{2n}\, z^n \sum_{r=0}^{\infty} \frac{(3x+4+2n)_r}{(1)_r} z^r$$

$$= (3x+4)_{2n}\, z^n (1-z)^{-3x-2n-4}$$

Substituting the value of $\alpha_n, \beta_n, \gamma_n$ and δ_n in (1.2), and after simplification we get result (3.1).

(ii) Now, we prove (3.2):

Taking $u_r = \frac{(1+x)_r(1+y)_r}{(1)_r}, v_r = 1$ $\alpha_r = \frac{(x)_r(y)_r}{(1)_r(-1)^r}$ and $\delta_r = \frac{1}{(\alpha)_r}$ in (1.1), we get

$$\beta_n = \sum_{r=0}^{n} \alpha_r u_{n-r} v_{n+r}$$

$$= \frac{(1+x)_n(1+y)_n}{(1)_n} \sum_{r=0}^{n} \frac{(-n)_r(x)_r(y)_r}{(-n-x)_r(-n-y)_r(1)_r}.$$

Now, making use of (2.4), we get

$$\beta_n = \frac{(1+x+y)_n(1+x)_n(1+y)_n}{(1)_n(1+x+y)_n}.$$

$$\gamma = \sum \delta_{r+n} u_r v_{r+2n}$$

$$= \sum_{r=0}^{\infty} \frac{1}{(\alpha)_{r+n}} \frac{(1+x)_r(1+y)_r}{(1)_r}$$

$$=\frac{1}{(\alpha)_n}\sum_{r=0}^{\infty}\frac{(1+x)_r(1+y)_r}{(1)_r(\alpha+n)_r}$$

$$=\frac{\Gamma(\alpha)\Gamma(\alpha-x-y-2)}{\Gamma(\alpha-x-1)\,\Gamma(\alpha-y-1)}\frac{(\alpha-x-y-2)_n}{(\alpha-x-1)_n(\alpha-y-1)_n}.$$

Now, putting of $\alpha_n,\beta_n,\gamma_n$ and δ_n in (1.2), and after simplification we get result (3.2).

(iii) The proof of (3.3) is as follows:

Taking $u_r=\frac{(1+y)_r}{(1)_r(1+x)_r}$, $v_r=1$, $\alpha_r=\frac{(-1)^r(y)_r}{(1+x)_r(1)_r}$ and $\delta_r=(\alpha)_r$. in (1.1),

we get

$$\beta_n=\sum_{r=0}^{n}\alpha_r u_{n-r}v_{n+r}$$

$$=\sum\frac{(-1)^r(y)_r}{(1+x)_r(1)_r}\frac{(1+y)_{n-r}}{(1)_{n-r}(1+x)_{n-r}}$$

$$=\frac{(1+y)_n}{(1+x)_n(1)_n}\sum_{r=0}^{n}\frac{(-1)^r(y)_r(1+y+n)_{-r}}{(1+x)_r(1)_r(1+n)_{-r}(1+x+n)_{-r}}$$

$$=\frac{(1+y)_n}{(1+x)_n(1)_n}\sum_{r=0}^{n}\frac{(-n)_r(-n-x)_r(y)_r}{(1)_r(-n-y)_r(1+x)_r}.$$

By making use of (2.5), we get:

$$\beta_n=\frac{1}{(1+x)_n}\frac{(1+x-y)_n(1+y)_m}{(1+x)_m(1)_m}.$$ and

$$\gamma_n=\sum_{r=0}^{\infty}\delta_{r+n}u_r v_{r+2n}$$

$$=\sum_{r=0}^{\infty}(\alpha)_{r+n}\frac{(1+y)_r}{(1)_r(1+x)_r}$$

$$=(\alpha)_n\,{}_2F_1\left[\begin{matrix}1+y,a+n\\ & ;1\\ 1+x\end{matrix}\right]$$

$$=(\alpha)_n\frac{\Gamma(1+x)\Gamma(x-y-\alpha-n)}{\Gamma(1+x-\alpha-n)\Gamma(x-y)}.$$

Now, putting the value of $\alpha_n, \beta_n, \gamma_n$ and δ_n in (1.2), and after simplification we get result (3.3).

(iv) The proof of (3.4) is as follows:

Taking $u_r = \frac{(x)_r (y)_r}{(1)_r}$, $v_r = 1$, $\alpha_r = \frac{(1+x)_r (1+y)_r}{(-1)^r (1)_n}$ and $\delta = \frac{1}{(z)_r}$. in (1.1), we get

$$\beta_n = \sum_{r=0}^{n} \alpha_r u_{n-r} v_{n+r}$$

$$= \frac{(x)_n (y)_n}{(1)_n} F\left[\begin{matrix} n, 1+x, 1+y \\ 1-n-x, 1-n-y \end{matrix}\right]$$

By making use of (2.6), we get:

$$\beta_n = (-1)^n (1+x+y)_n \frac{(1+x)_m (1+y)_m}{(1+x+y)_m (1)_m}$$

$$\gamma_n = \sum_{r=0}^{\infty} \delta_{r+n} u_r v_{r+2n}$$

$$= \sum_{r=0}^{n} \frac{1}{(z)_{r+n}} \frac{(x)_r (y)_r}{(1)_r}$$

$$= \frac{1}{(z)_n} \frac{\Gamma(z+n)\Gamma(z+n-x-y)}{\Gamma(z+n-x)\Gamma(z+n-y)}$$

Now, putting the value in (1.2), and after simplification we get result (3.4)

(v) The proof of (3.5) is as follows:

Taking $u_r = \frac{(1+x)_r}{(1)_r (1+2x)_r}$, $v_r = (1)^r$, $\alpha_r = \frac{(y)_r (-1)^r}{(2y+1)_r (1)_r}$ and $\delta_r = (\alpha)_r$.

$$\beta_n = \sum \alpha_r u_{n-r} v_{n+r}$$

$$= \sum_{r=0}^{n} \frac{(y)_r (-1)^r}{(2y+1)_r (1)_r} \frac{(1+x)_{n-r}}{(1)_{n-r} (1+2x)_{n-r}}.$$

By making use of (2.7), we get:

$$\beta_n = \frac{(1+x+y)_n (1+x)_m (1+y)_m}{(1+2x)_n (1+2y)_n (1+x+y)_m (1)_m}.$$

$$\gamma_n = \sum_{r=0}^{n} \delta_{r+n} u_r v_{r+2n}$$

$$=\sum_{r=0}^{\infty}(\alpha)_{r+n}\frac{(1+x)_r}{(1)_r(1+2x)_r}(1)^{r+2n}$$

$$=(\alpha)_n\,{}_2F_1\left[\begin{matrix}\alpha+n,1+x\\ & ;1\\ 1+2x\end{matrix}\right]$$

$$=\frac{\Gamma(1+2x)(\alpha)_n\Gamma(x-\alpha)(x-\alpha)_n}{\Gamma(x)\Gamma(1+2x-\alpha)(1+2x-\alpha)_n}.$$

Substituting the value of $\alpha_n,\beta_n,\gamma_n$ and δ_n in (1.2), and after simplification we get result (3.5).

(vi) The proof of (3.6) is as follows:

Taking $u_r=\frac{(x)_r}{(1)_r(1+2x)_r}$, $v_r=(1)^r$, $\alpha_r=\frac{(-1)^r(1+y)_r}{(1+2y)_r(1)_r}$ and $\delta_r=(\alpha)_r$

$$\beta_n=\sum_{r=0}^{n}\alpha_r u_{n-r}v_{n+r}$$

$$=\sum_{r=0}^{n}\frac{(-1)^r(1+y)_r}{(1+2y)_r(1)_r}\frac{(x)_{n-r}}{(1)_{n-r}(1+2x)_{n-r}}1^{n+r}.$$

By making use of (2.8), we get:

$$\beta_n=\frac{(-1)^n(1+x+y)_n(1+x)_m(1+y)_m}{(1+2x)_n(1+2y)_n(1+x+y)_m(1)_m}.$$

$$\gamma_n=\sum_{r=0}^{n}\delta_{r+n}u_r v_{r+2n}$$

$$=\sum_{r=0}^{\infty}(\alpha)_{r+n}\frac{(x)_r}{(1)_r(1+2x)_r}1^{r+2n}.$$

$$=\frac{(\alpha)\Gamma(1+2x)\Gamma(1+x-\alpha)(1+x-\alpha)_n}{\Gamma(1+x)\Gamma(1+2x-\alpha)(1+2x-\alpha)_n}.$$

Now, substituting the value of $\alpha_n,\beta_n,\gamma_n$ and δ_n in (1.2), and after simplification we get result (3.6).

(vii) The proof of (3.7) is as follows:

Taking $u_r=\frac{1}{(1)_r}$, $v_r=(1+2x+2y)_r$, $\alpha_r=\frac{(x)_r}{(1+x+y)_r(1+2x)_r(1)_r}$ and $\delta_r=z^r$.

$$\beta_n = \sum_{r=0}^{n} \alpha_r u_{n-r} v_{n+r}$$

$$= \sum_{r=0}^{n} \frac{(x)_r}{(1+x+y)_r (1+2x)_r (1)_r} \frac{(1+2x+2y)_{n+r}}{(1)_{n-r}}.$$

$$= \frac{(1+2x+2y)_n}{(1)_n} \sum_{r=0}^{n} \frac{(x)_r (1+2x+2y+n)_r (-n)_r}{(1+x+y)_r (1+2x)_r (1)_r (-)^r}.$$

By making use of (2.9), we get:

$$\beta_n = \frac{(1+2x+2y)_n (1+x)_m (1+y)_m}{(1+2x)_n (1+x+y)_m (1)_m}.$$

$$\gamma_n = \sum_{r=0}^{n} \delta_{r+n} u_r v_{r+2n}$$

$$= \sum_{r=0}^{n} z^{r+n} \frac{1}{(1)_r} (1+2x+2y)_{r+2n}.$$

$$= z^n (1+2x+2y)_{2n} \sum_{r=0}^{\infty} \frac{(1+2x+2y+2n)_r}{(1)_r} z^r.$$

$$= z^n (1+2x+2y)_{2n} (1-z)^{-1-2x-2y-2n}.$$

Substituting the value of $\alpha_n, \beta_n, \gamma_n$ and δ_n in (1.2), and after simplification we get result (3.7).

(viii) The proof of (3.8) is as follows:

Taking $u = \frac{1}{(1)_r}$, $v_r = (1+2x+2y)_r$, $\alpha_r = \frac{(-)^r (1+x)_r}{(1+x+y)_r (1+2x)_r (1)_r}$ and $\delta_r = z^r$.

$$\beta_n = \sum_{r=0}^{n} \alpha_r u_{n-r} v_{n+r}$$

$$= \sum_{r=0}^{n} \frac{(-)^r (1+x)_r}{(1+x+y)_r (1+2x)_r (1)_r} \frac{(1+2x+2y)_{n+r}}{(1)_{n-r}}.$$

By making use of (2.10), we get:

$$\beta_n = \frac{(1+2x+2y)_n (-)^n (1+x)_m (1+y)_m}{(1+2x)_n (1+x+y)_m (1)_m}.$$

$$\gamma_n = \sum_{r=0}^{n} \delta_{r+n} u_r v_{r+2n}$$

$$= \sum_{r=0}^{\infty} z^{r+n} \frac{1}{(1)_r} (1+2x+2y)_{r+2n}$$

$$= (1+2x+2y)_{2n}\, z^n\, (1-z)^{-1-2x-2y-2n}.$$

Substituting the value of $\alpha_n, \beta_n, \gamma_n$ and δ_n in (1.2), and after simplification we get result (3.8).

(ix) The proof of (3.9) is as follows:

Taking $u = \frac{1}{(1)_r}$, $v_r = (2+b+2x)_r$, $\alpha_r = \frac{(-)^r (1+x)_r}{(1+\frac{1}{2}b+x)_r (2+2x)_r (1)_r}$ and $\delta_r = z^r$.

$$\beta_n = \sum_{r=0}^{n} \alpha_r u_{n-r} v_{n+r}$$

$$= \sum_{r=0}^{n} \frac{(-)^r (1+x)_r}{(1+\frac{1}{2}b+x)_r (2+2x)_r (1)_r} \frac{(2+b+2x)_{n+r}}{(1)_{n-r}}.$$

By making use of (2.11), we get:

$$\beta_n = \frac{(-1)^n (\frac{3}{2}+\frac{b}{2}+x)_m (1+\frac{b}{2})_m}{(\frac{3}{2}+x)_m (1)_m}.$$

$$\gamma_n = \sum_{r=0}^{n} \delta_{r+n} u_r v_{r+2n}$$

$$\gamma_n = \sum_{r=0}^{\infty} z^{r+n} \frac{1}{(1)_r} (2+b+2x)_{r+2n}$$

$$= z^n (2+b+2x)_{2n} (1-z)^{-2-b-2x-2n}.$$

Substituting the value of $\alpha_n, \beta_n, \gamma_n$ and δ_n in (1.2), and after simplification we get result (3.9).

(x) The proof of (3.10) is as follows:

Taking $u_r = \frac{1}{(1)_r}$, $v_r = (1+a)_r$, $\alpha_r = \frac{(x)_r (\frac{1}{2}x-\frac{1}{2})_r (-)^r}{(x-1)_r (x+1)_r (1)_r (\frac{1}{2}+\frac{a}{2})_r}$. and $\delta_r = z^r$.

$$\beta_n = \sum_{r=0}^{n} \alpha_r u_{n-r} v_{n+r}$$

$$\beta_n = \sum_{r=0}^{n} \frac{(x)_r (\tfrac{1}{2}x - \tfrac{1}{2})_r (-)^r}{(x-1)_r (x+1)_r (1)_r (\tfrac{1}{2} + \tfrac{a}{2})_r} \frac{1}{(1)_{n-r}} (1+a)_{n+r}.$$

By making use of (2.12), we get:

$$\beta_n = \frac{(1+a-x)_n (1+\tfrac{1}{2}a)_m (\tfrac{1}{2}+\tfrac{1}{2}a)_m}{(1+x)_n (1)_m (\tfrac{1}{2}+\tfrac{1}{2}a-\tfrac{1}{2}x)_m}.$$

$$\gamma_n = \sum_{r=0}^{n} \delta_{r+n} u_r v_{r+2n}$$

$$\gamma_n = \sum_{r=0}^{\infty} z^{n+r} \frac{1}{(1)_r} (1+a)_{r+2n}.$$

$$= (1+a)_n (1+a+n)_n z^n (1-z)^{-1-a-2n}.$$

Substituting the value of $\alpha_n, \beta_n, \gamma_n$ and δ_n in (1.2), and after simplification we get result (3.10).

5. CONCLUSION

In this paper many formulae for Hypergeometric functions can be obtained using Bailey's transform and truncated series

REFERENCES

1. Agarwal, R. P. (1996): Resonance of Ramanujan's mathematics, volume-I, New Age International limited, New Delhi.
2. Bailey,W.N. (1974): Some Identities in combinatory analysis, Proc.London Math.Soc.,49,421-435.
3. Denis,R.Y.,Singh,S.N. and Singh,S.P. (2007): On certain transformation involving truncated q-series, J. Indian Math.Soc., 74, 1-2,115-125.
4. Singh U. B. (1949): A note on a transformation of Bailey, Q. J. Math. Oxford, 2(45), 111-116.
5. Sharma M.K., Bansal K.K.(2015): "Fuzzy Analysis Of Shortest Job First" International Journal of Engineering Research & Management Technology Vol.2(3)
6. Slater, L. J. (1966): Generalized hypergeometric Series, Cambridge University Press, London (U. K).
7. Srivastava, Pankaj. (2004): A note on Biley's transform, South East Asian J. Math.Sci. , (2) 2, 9-14.
8. Srivastava, P. and Mohan, R. (2011): Certain transformation formulae for polybasic hyper geometric series, vol. 2011, Article ID 248519, ISRN Algebra.
9. Verma, A. and Jain, V. K., (1980): Some summation formulae of basic hyper geometric series, Indian J. Pure Math., 11(8), 1021-1038.
10. Warnaar, S. O. (2001): 50 years of Bailey's lemma, in Algebraic Combinatorics and Applications, A. Betten et al. Eds., Springer, Berlin, 333-347.